Calculations using standard form.

To **add or subtract** two numbers in scientific notation, you first need to convert them to the same power.

For example: $3 \times 10^4 + 2 \times 10^5$
$= 3 \times 10^4 + 20 \times 10^4$
$= 23 \times 10^4$
$= 2.3 \times 10^5$

you could also write this:

30000
+ 200000
230000

To **multiply** numbers with the same base, add the exponents

$$a^b \times a^c = a^{b+c}$$

For example: $10^4 \times 10^5 = 10^9$

Another example: $7 \times 10^4 \times 2 \times 10^{-6}$
$= 14 \times 10^{4-6}$
$= 14 \times 10^{-2}$
$= 1.4 \times 10^{-1}$

What is the **power of a power**?

$$\left(a^b\right)^c = a^{bc}$$

For example: $(10^4)^{-5} = \frac{1}{\left(10^4\right)^5} = \frac{1}{10^4 \times 10^4 \times 10^4 \times 10^4 \times 10^4} = \frac{1}{10^{20}} = 10^{-20}$

Another example: $(3 \times 10^{-6})^3 = 3^3 \times (10^{-6})^3 = 27 \times 10^{-18}$

To **divide** numbers with the same base, subtract the exponents

$$\frac{a^b}{a^c} = a^{b-c}$$

For example: $\frac{10^{10}}{10^6} = 10^4$

Another example: $\frac{5 \times 10^{-4}}{2 \times 10^{-6}} = 2.5 \times 10^{-4--6} = 2.5 \times 10^2$

But what if b – c gives zero?

$$a^0 = 1$$

If b - c is zero, then the exponents were the same and this is the same as dividing a number by itself which of course gives one.

For example: $\frac{10^5}{10^5} = 10^{5-5} = 10^0 = 1$ this is the same calculation as $\frac{100000}{100000} = 1$

Fractional powers.

Once we understand that $10^a \times 10^b = 10^{a+b}$ then it becomes clear that the values for a and b do not need to be integers. For example, consider the following,

$$10^{0.5} \times 10^{0.5} = 10^{0.5+0.5} = 10^1 = 10$$

This is the same as writing: $10^{1/2} \times 10^{1/2} = 10^{1/2+1/2} = 10$

which is the same as: $\sqrt{10} \times \sqrt{10} = 10$

Similarly $10^{1/3} \times 10^{1/3} \times 10^{1/3} = 10$

is the same as writing: $\sqrt[3]{10} \times \sqrt[3]{10} \times \sqrt[3]{10} = 10$

The symbols √ and ³√ are typically only used for square roots and cubed roots i.e. $10^{1/2}$ and $10^{1/3}$ respectively. Otherwise we just use the decimal notation eg $10^{0.793}$.

Fractional powers follow all the same rules as integer powers.

multiplying $10^a \times 10^b = 10^{a+b}$	example:	$10^{0.3} \times 10^{0.8} = 10^{1.1}$
dividing $\frac{10^a}{10^b} = 10^{a-b}$	example:	$\frac{10^{0.3}}{10^{0.9}} = 10^{0.3-0.9} = 10^{-0.6}$
powers of powers $(10^a)^b = 10^{a \times b}$	example:	$(10^{0.3})^{0.5} = 10^{0.15}$
For addition and subtraction we must convert to the same power, so:	$10^{-6.3} + 10^{-6.9}$ $= 5.01187 \times 10^{-7} + 1.2589 \times 10^{-7}$ $= 6.27077 \times 10^{-7}$	

Exercises

1) If the weight of a typical rat brain is 2 g and the brain contains roughly 2×10^8 neurons, how many neurons would be contained in a 10 µg sample?

2) If 6×10^{23} carbon atoms weigh 12g, how much would 1 carbon atom weigh?

3) An experimental robot is able to dispense accurately 2.0 pL volumes. If the robot had 9200 tubes in which it had to dispense its solution, what total volume would it need (use an appropriate unit and express your answer to two significant figures)?

4) If you weighed out 1.3 mg of drug from a bottle containing 0.4 g, how much drug remains in the bottle?

5) If one side of a cube-shaped cell had a length = 2×10^{-5} m, what is its volume in m^3?

6) Light microscopy can resolve detail to 0.2 μm, a limitation imposed by the wavelength of light not by the quality of lenses. A scanning electron microscope can resolve details down to 3 nm. What is the difference in resolution between these two microscopy techniques? Give your answer in nm.

7) The E. coli genome is 4.8 million base pairs compared to a human genome of 6 billion base pairs (a billion is a thousand times a million). How many times larger is the human genome than that of E. coli?

8) The total number of bacteria in a 70 kg adult has been estimated to be 3.8×10^{13}. If the mass of a single bacterium is 10^{-12} g, what is the mass of the bacteria in an adult?

9) There are about 2×10^{12} lymphocytes in the human body, making the immune system comparable in cell mass to the liver or brain. There are approximately 1×10^{13} cells in the human body, estimate what proportion of human body cells are lymphocytes? Don’t use a calculator and show your working.

Answers

(1) $$\frac{2\times10^8}{2g}=\frac{x}{10\times10^{-6}g}$$

multiply both sides by 10 x 10^{-6} g: $\frac{2\times10^8}{2g}\times10\times10^{-6}g=x$

$$x=10\times10^2=1000$$

There would be 1000 neurons ina 10 µg sample.

(2) $\frac{12g}{6\times10^{23}}=2\times10^{-23}g$ *One carbon atom would weigh 2 x 10^{-23} g*

(3) 9200 x 2.0 x 10^{-12} L = 9.2 x 10^3 x 2.0 x 10^{-12} L = 18.4 x 10^{-9} L.
To 2 significant figures this is 18 nL.

(4) 0.4 g = 400 mg, so 400 mg – 1.3 mg = 398.7 mg. This is the amount remaining.
Alternatively you could write 0.4 g – 0.0013 g = 0.3987 g.

5) Volume = $(2 \times 10^{-5}\ m)^3$
= $2^3 \times (10^{-5})^3\ m^3$
= $8 \times 10^{-15}\ m^3$.

6) 0.2 µm = 200 nm.
200 nm – 3 nm = 197 nm

7) $\frac{6 \times 10^9}{4.8 \times 10^6}$ = 1.25 x 10^3 or 1250 times larger

8) 3.8 x 10^{13} x 1 x 10^{-12} = 3.8 x 10^1 = 38 g

9) $\frac{2 \times 10^{12}}{1 \times 10^{13}}$ = 2 x 10^{-1} = 0.2 or 20%